Euler's method

In Taylor's series method, we obtain approximate solutions of the initial value problem $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ as a power series in x, and the solution can be used to compute y numerically specified value x near of y.

In Euler's method, we compute the values of y for $x_i = x_0 + ih$, with a step size $h > 0$,

(ie) $y_i = y(x_i)$ where $x_i = x_0 + ih$,

$i = 1, 2, 3 \ldots\ldots$

Formula:

neglecting the terms with h^2 and higher powers of h, we get

$$y_1 = y_0 + h f(x_0, y_0)$$

III^{elly} $y_2 = y_1 + h f(x_1, y_1)$

In general $y_{m+1} = y_m + h f(x_m, y_m)$

Problem 1

Prob. 1

Using Euler's method, compute y in the range $0 \le x \le 0.5$, if y satisfies $\frac{dy}{dx} = 3x + y^2$, $y(0) = 1$.

Soln: $f(x, y) = 3x + y^2$, $y(0) = 1$.

$x_0 = 0$, $y_0 = 1$.

By Euler's method,

$$y_{n+1} = y_n + h f(x_n, y_n),$$

$n = 0, 1, 2 \ldots$

Choosing $h = 0.1$, we compute the values of y.

$$y(0.1) = y_1 = y_0 + h f(x_0, y_0)$$

$$y_1 = y_0 + 0.1 (3x_0 + y_0^2)$$

$$y_1 = 1 + 0.1 [3(0) + 1^2]$$

$$\boxed{y_1 = 1.1}$$

$$y(0.2) = y_2 = y_1 + h f(x_1, y_1)$$

$$y_2 = y_1 + 0.1 [3x_1 + y_1^2]$$

$$y_2 = 1.1 + 0.1 [3(0.1) + (1.1)^2]$$

$$\boxed{y_2 = 1.251}$$

$$y(0.3) = y_3 = y_2 + h f(x_2, y_2)$$

$$y_3 = 1.251 + 0.1\left[3(0.2) + (1.251)^2\right]$$

$$\boxed{y_3 = 1.4675}$$

$$y(0.4) = y_4 = y_3 + h f(x_3, y_3)$$

$$y_4 = 1.4675 + 0.1\left[3(0.3) + (1.4675)^2\right]$$

$$\boxed{y_4 = 1.7728}$$

$$y(0.5) = y_5 = y_4 + h f(x_4, y_4)$$

$$y_5 = 1.7728 + 0.1\left[3(0.4) + (1.7728)^2\right]$$

$$\boxed{y_5 = 2.2071}$$

Prob: 2

Using Euler's method, solve $y' = x + y + xy$, $y(0) = 1$. compute $y(1.0)$ with $h = 0.2$.

Soln: Given $f(x, y) = x + y + xy$

$y(0) = 1$.

$x_0 = 0$, $y_0 = 1$.

W.K.T $y_{n+1} = y_n + h f(x_n, y_n)$

Let $h = 0.2$

$$y_1 = y_0 + h f(x_0, y_0)$$

$$y(0.2) = y_0 + 0.2 [x_0 + y_0 + x_0 y_0]$$

$$= 1 + 0.2 [0 + 1 + 0]$$

$$\boxed{y_1 = y(0.2) = 1.2}$$

$$y_2 = y_1 + h f(x_1, y_1)$$

$$y(0.4) = y_1 + (0.2) [x_1 + y_1 + x_1 y_1]$$

$$= 1.2 + 0.2 [0.2 + 1.2 + 0.2 \times 1.2]$$

$$y(0.4) = 1.2 + (0.2) [0.2 + 1.2 + 0.24]$$

$$\boxed{y_2 = y(0.4) = 1.528}$$

$$\boxed{y_2 = y(0.4) = 1.528}$$

$$y_3 = y_2 + h\, f(x_2, y_2)$$

$$y(0.6) = y_2 + (0.2)[x_2 + y_2 + x_2 y_2]$$

$$y(0.6) = 1.528 + (0.2)[0.4 + 1.528 + 0.4 \times 1.528]$$

$$= 1.528 + (0.2)[0.4 + 1.528 + 0.6112]$$

$$\boxed{y_3 = y(0.6) = 2.0358}$$

$$y(0.8) = y_4 = y_3 + h\, f(x_3, y_3)$$

$$y_4 = y_3 + (0.2)[x_3 + y_3 + x_3 y_3]$$

$$y_4 = 2.0358 + (0.2)[0.6 + 2.0358 + 0.6 \times 2.0358]$$

$$y_4 = 2.0358 + (0.2)[0.6 + 2.0358 + 1.22148]$$

$$\boxed{y_4 = y(0.8) = 2.8072}$$

$$y(1.0) = y_5 = y_4 + hf(x_4, y_4)$$

$$y_5 = y_4 + (0.2)[x_4 + y_4 + x_4 y_4],$$

$$y_5 = 2.8072 + 0.2\,[0.8 + 2.8072$$

$$y_5 = 2.8072 + 0.2[0.8 + \ldots + 0.8 \times 2.8072]$$

$$y_5 = 2.8072 + 0.2[0.8 + 2.8072 + 2.24576]$$

$$\boxed{y(1.0) = y_5 = 3.9778}$$